Gamma-ray Spectrometry

The basic equation for gamma-ray spectrometry

Mikael Hult (Docent, PhD, Lic Eng, MSc)

European Commission – Joint Research Centre - Institute for Reference Materials and Measurements

Mikael.Hult@ec.europa.eu

E&I, Belgrade, Serbia

European Commission

Obligatory slide

Slides 3 to end are the intellectual property of Docent Mikael Hult (EC-JRC-IRMM).

"Unauthorised reproduction constitutes a copyright infringement and may lead to prosecution or civil proceedings"

European Commission

Basic Equation

$$C = AP_\gamma t\varepsilon$$

$$A = \frac{C}{t\varepsilon P_\gamma}$$

Peak Count

Gamma-ray emission probability

FEP (Full Energy Peak) efficiency

Measurement time (live time)

$$A = \frac{C_{TOT} - C_{Peak}^{Bkg} - C_{Continuum}}{\varepsilon_{REF}^{Exp} \frac{\varepsilon_{Sample}^{MC}}{\varepsilon_{REF}^{MC}} P_\gamma} e^{\lambda t_d} \frac{\lambda}{(1 - e^{-\lambda t_m})} K_1 K_2 K_3$$

Reference sample similar to the measured one

Correction factor from e.g. EGS4 MonteCarlo code

K_1 = summing correction

K_2 = Branching correction

K_3 = Equilibrium correction

t_d = decay time (to a reference date)
t_m = measurement live time

Uncertainties on efficiency down to ~ 1%

Combined activities from several gamma-rays and from several daughters to activity for one radionuclide

European Commission

Basic Equation – Time corrections

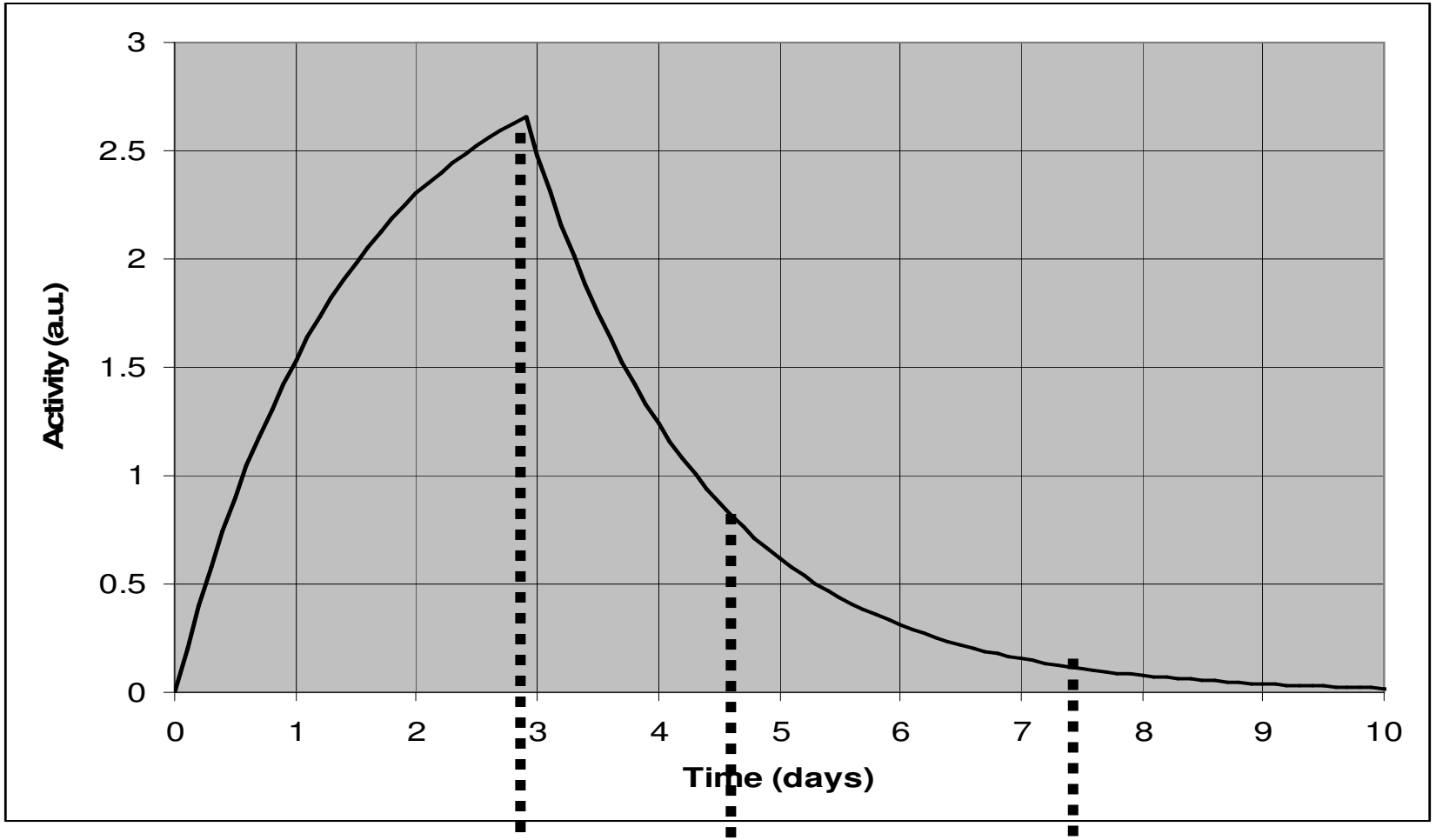
Decay during measurement and decay from reference date

$$\frac{1}{t_m} \Rightarrow \frac{\lambda}{(1-e^{-\lambda t_m})}$$

Decay from reference date $e^{\lambda t_d}$

Basic Equation – Time corrections

European Commission



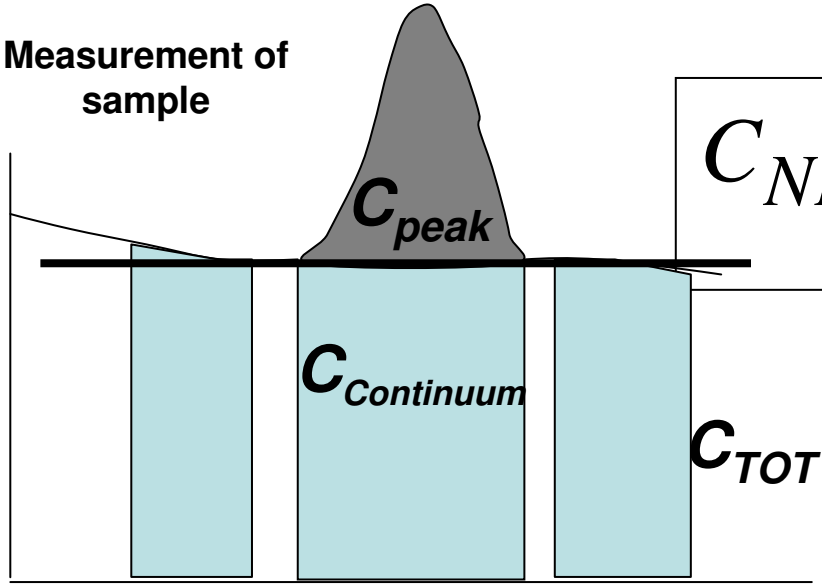
$t_a = t_i =$ activation time

$t_d =$ decay time

$t_m =$ measurement time (real time)

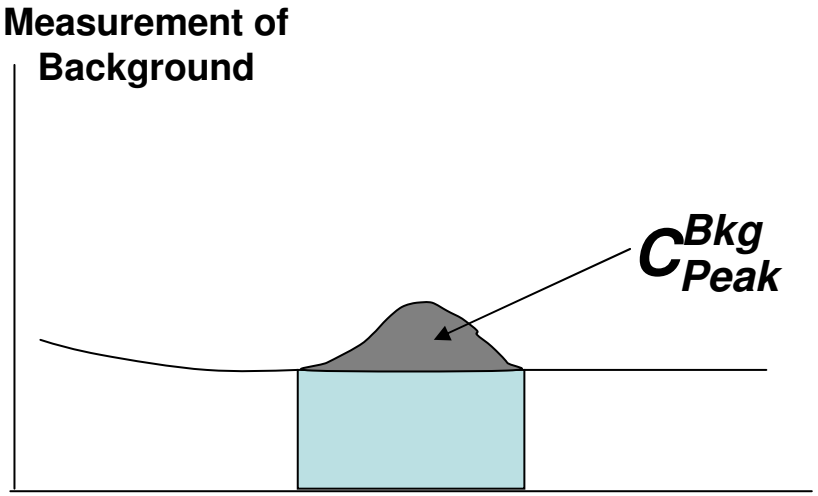
Basic Equation – Peak Counts

European Commission



$$C_{NET} = C_{TOT} - C_{Peak}^{Bkg} - C_{Continuum}$$

$$C_{TOT} = C_{Continuum} + C_{peak}$$



Basic Equation – Peak Counts – peak fitting

Different algorithms for different data analysis software

- Genie-2000 (Canberra)
- InterWinner (Eurisys)
- Gamma-Vision (Ortec)
- Gamma-W (Dr. Westmaier)
- Fitzpeaks
- HYPERMET

+ many other more or less home-made ones

A Golden Rule

- **Don't trust an automatic fit. Use visual inspection whenever possible!**

Common problems:

- **unexpected Interfering peak(s),**
- **Errors in nuclear data library,**
- **erroneous settings used,**
- **bad peak shape due to unstable electronics during measurements, etc.**

The best QC is to visually inspect spectra!

Basic Equation – Peak Counts – peak fitting

European Commission

- Start with a good energy calibration and a good calibration of the FWHM……and good quality data (Gaussian peaks)
- Ideally a full-energy peak can be represented by a Gaussian function
- In practice there is tailing on the low-E side of the peak (also at hi-E side at e.g. high count rates)
- Note different shapes for X-rays peaks and annihilation peak

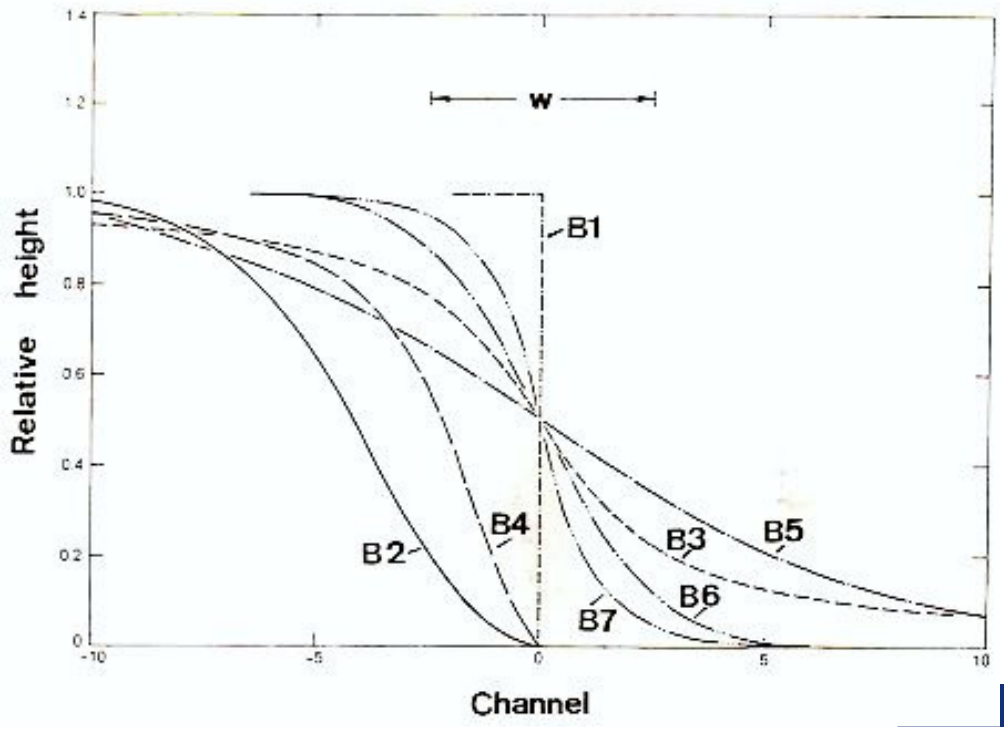
Basic Equation – Peak Counts – peak fitting

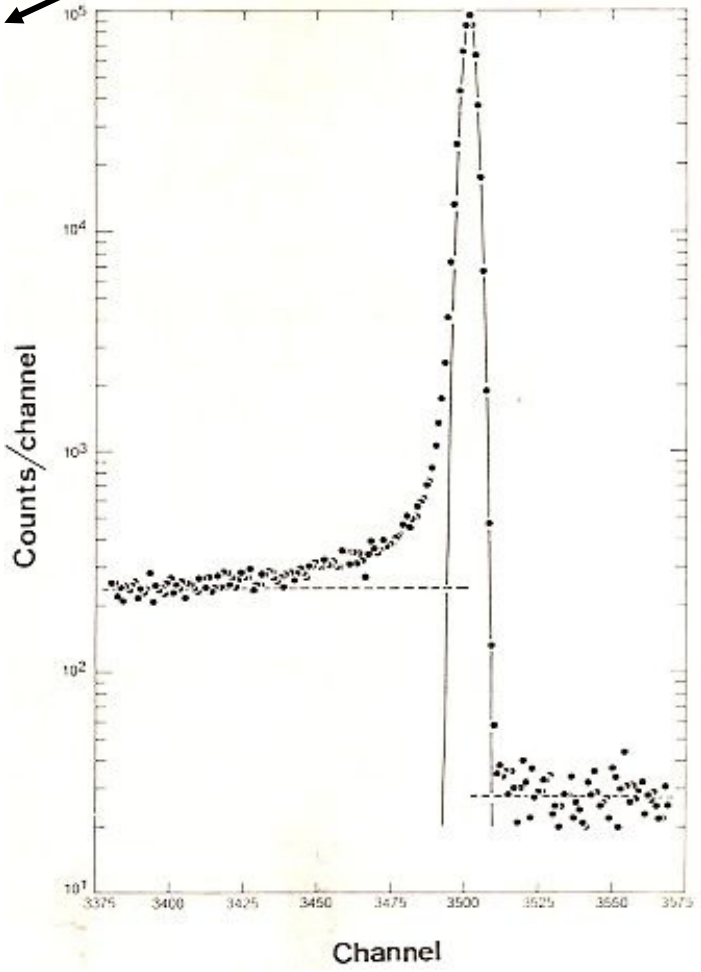
European Commission

Most common fits include

1. A Gaussian (main feature)
2. A liner background with step-function
3. A linear low-E tail

Mainly these varies between software



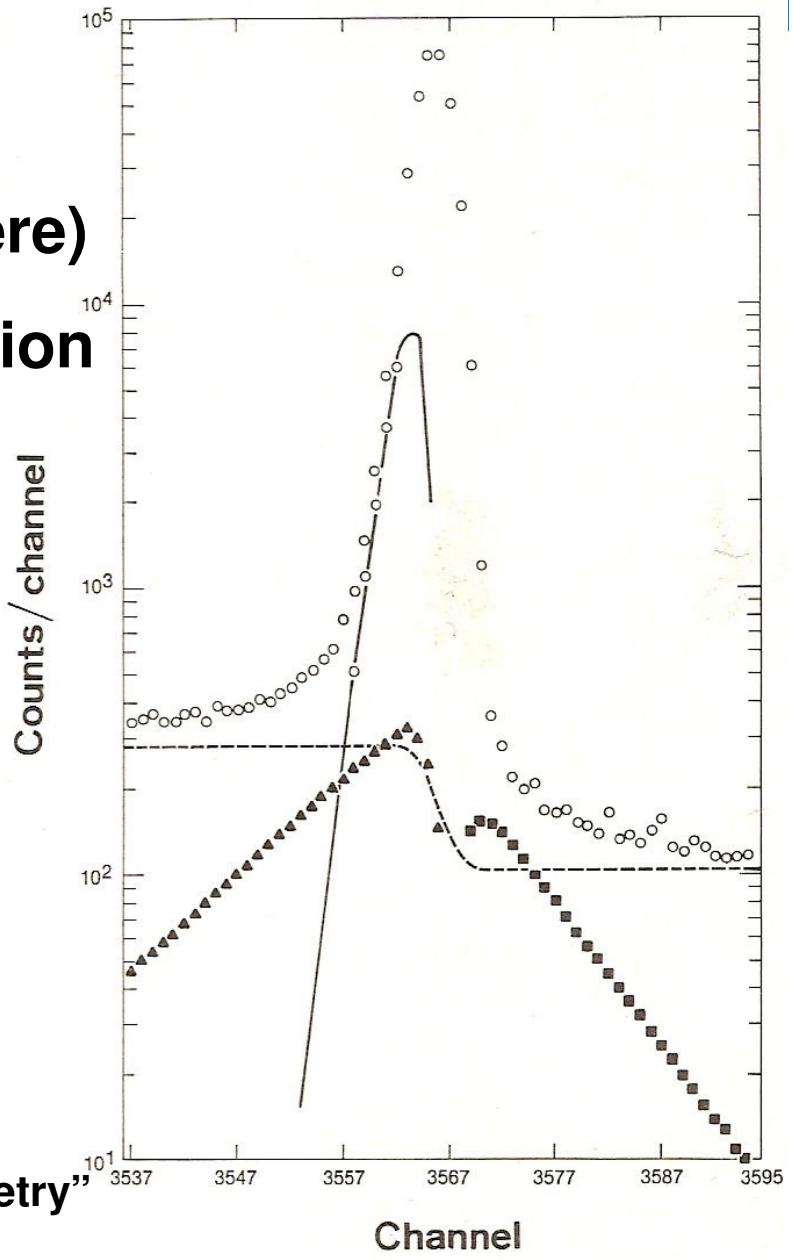


Basic Equation – Peak Counts – peak fitting

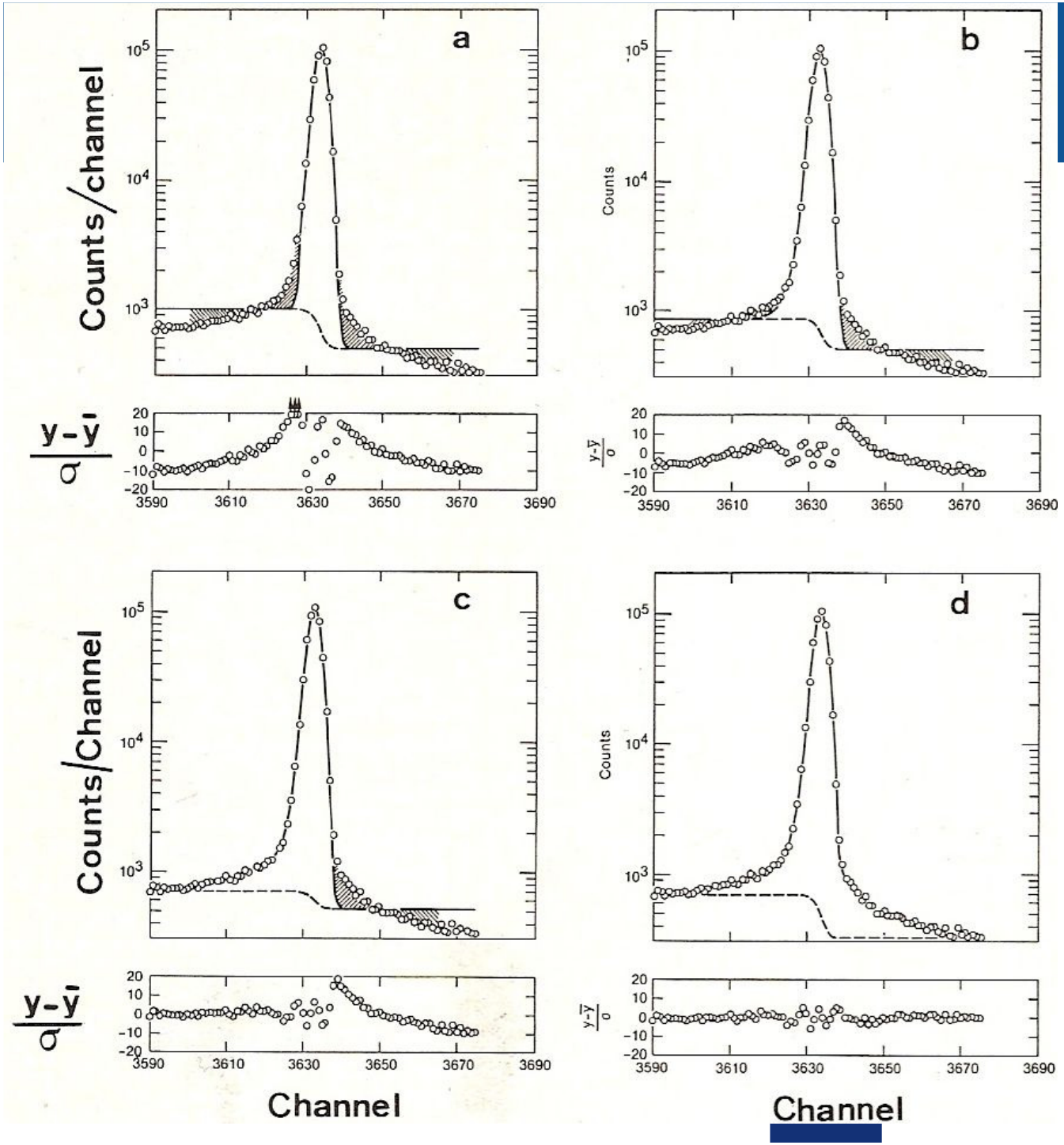
European Commission

More complex fits include

1. Gaussian (main feature-not shown here)
2. Constant background with step-function
3. Long term linear low-E tail
4. Short term low-E-tail
5. Hi-E tail



From: Debertin and Helmer, "X-ray and gamma-ray spectrometry"

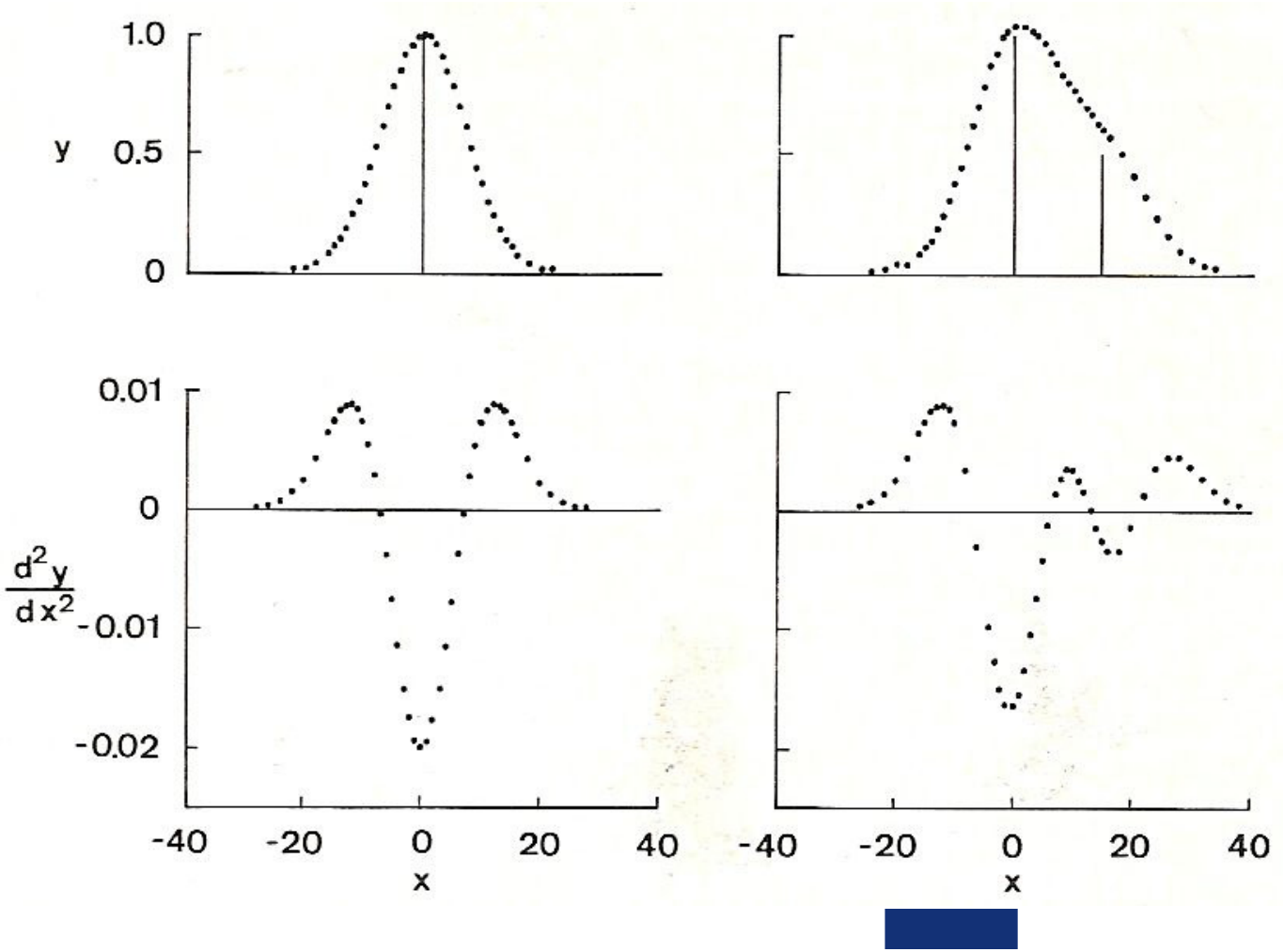


Less tailing with "new generation" HPGe-detectors!

Important to have "nice" well separated peaks to determine parameters for tailing

European Commission

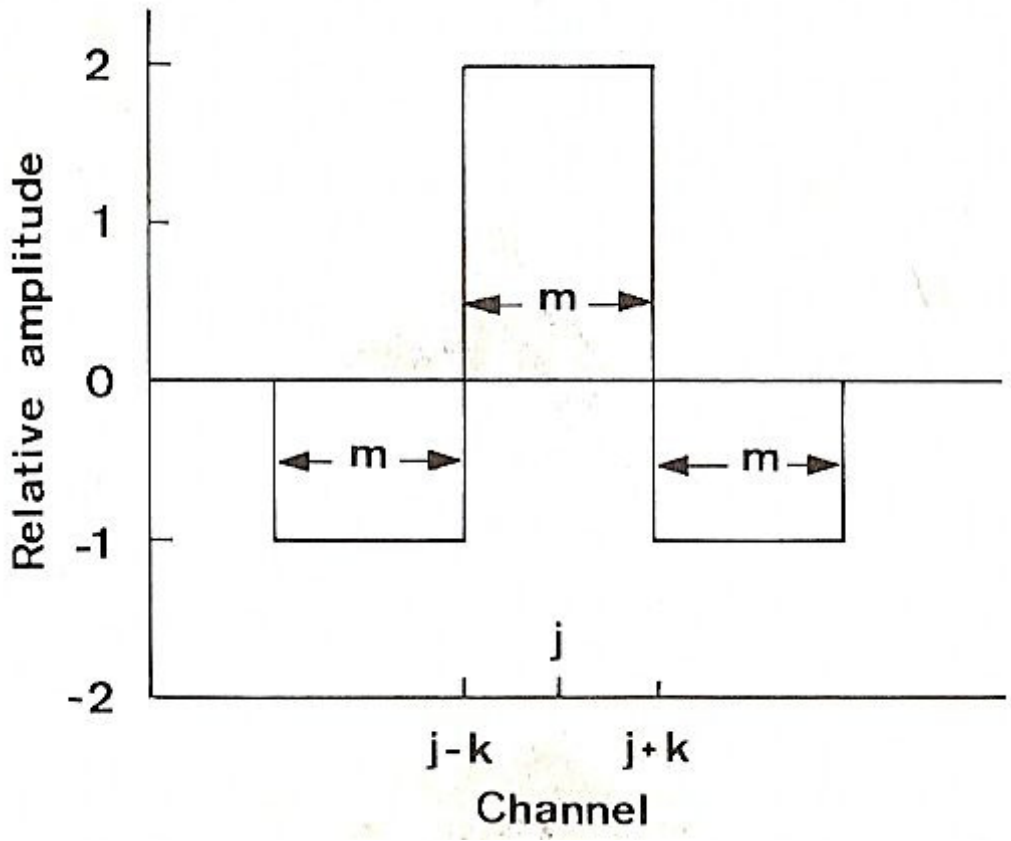
Automatic Peak Search – 2nd derivative



- Smoothed data
- apply statistical selection criteria
- => can distinguish single peak/doublets/Compton edges etc.

European Commission

Automatic Peak Search – Filter



- **The simplest filter shown here**
- **Calculate C**

$$C_j = \sum_{i=j-k}^{j+k} 2y_i - \sum_{i=j-k-(2k+1)}^{j-k-1} y_i - \sum_{i=j+k+1}^{j+k+(2k+1)} y_i$$

- **C > given value => peak detected**

Basic Equation – Decay Data

Referring to presentation by Vidmar (Bé)!

$\underline{P\gamma}$ (gamma-ray emission probability) and

$\underline{\lambda}$ (Decay constant) can vary significantly between different sources of data

Pay attention to equilibrium between mother-daughter, which decay constant (half-life) to use. Not automatically correct in commercial software!

And library data may not be up to date and not from DDEP.

Basic Equation – Efficiency

Measure FEP efficiency in a calibration source with several gamma-rays

Fit an empirical (or semiempirical) function to the data

If the "real sample" is different from the calibration source in shape, mass or composition – calculate efficiency correction using an efficiency transfer code

And… Note that time corrections usually needed !!!

$$\varepsilon = \frac{C}{tAP_\gamma}$$

BUT: Beware of coincidence summing corrections!!!

Basic Equation – Efficiency

European Commission

Lots of articles on "suitable" mathematical expressions, some general others very specific for a certain geometry and detector

α_1: adjustable parameter

α_2 and α_3, Depends on crystal size

Loss of Compton scattered photons

Semi-empirical Early attempt (1966):

$$\varepsilon(E) = a_1[\tau(E) + \sigma(E)\alpha_2 e^{-\alpha_3 E}]$$

Photoelectric cross-section of Ge

Compton cross-section of Ge

Only useful for small detectors and energies from 500 keV to 1500 keV

Empirical attempts with polynomial liner or non-linear functions dominate today, e.g.:

$$\varepsilon = \frac{1}{E}\sum_{i=1}^{8} a_i (\log E / E_0)^{i-1}$$

$$\log \varepsilon = \sum_{i=1}^{n} a_i (\log E / E_0)^{i-1}$$

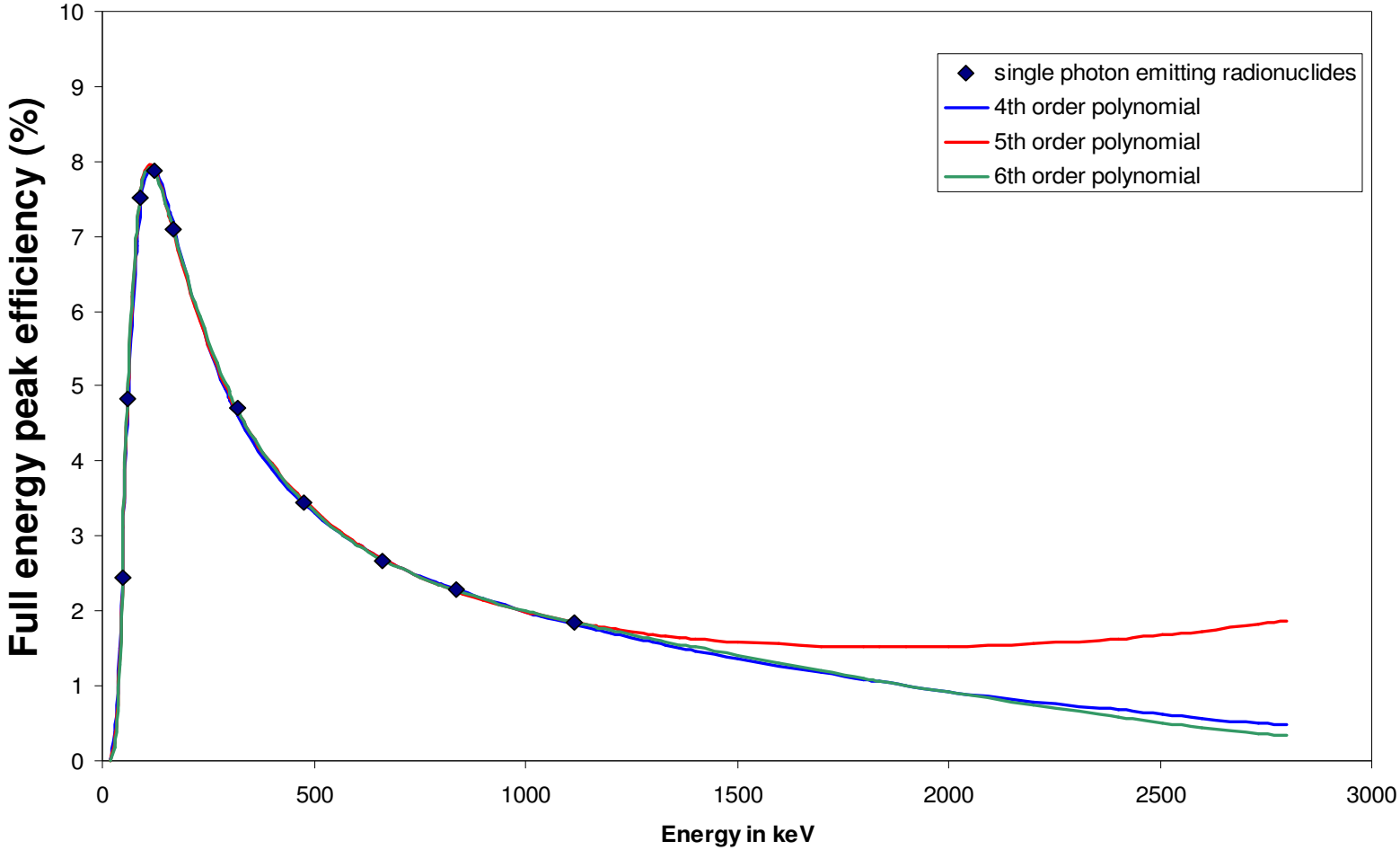
Used e.g. in Genie 2000

But needs a second polynomial to handle the low energy part (below ~200 keV)

Basic Equation – Efficiency

European Commission

Full energy peak efficiency for Pb-210, Am-241, Cd-109, Co-57, Be-7, Cs-137, Mn-54, Zn-65; 100 ml; p-type detector



Several codes for calculating efficiency transfer exists: EFFTRAN, GESPECOR, DETEFF, ETNA,……

Efficiency corrections can be calculated using general purpose Monte Carlo codes like: GEANT, PENELOPE, EGS4 and MCNP

$$\varepsilon = \varepsilon_{REF}^{Exp} \frac{\varepsilon_{Sample}^{MC}}{\varepsilon_{REF}^{MC}}$$

Absolute ε-values from Monte Carlo codes

It is possible to use the absolute efficiency calculated using a Monte Carlo code but great care should be taken since:

- **The computer model of the sample and detector can never describe reality in detail**
- **It is easy to make a small mistake**
- **The model might well describe the reference sample used for setting up the model – but might fail when the geometry changes.**
- **It is difficult to assess the uncertainty of a Monte Carlo calculation**

European Commission

Realisation of Calculations

1. **Making full use of commercial software**
   - **Most suited for routine analysis (analysis of many samples of the same type (matrix, mass, radionuclides)**
   - **Drawbacks: User has not full control of all calculations.**

2. **Home-made specially designed software**
   - **Takes long to implement**

3. **Making tailor-made calculation sheets in general purpose codes like MS Excel**
   - **Can get “messy” unless care is taken to “program cleanly”**

Const. T, etc.

Stable electricity

Good quality data

E-cal.
FWHM cal.

Peak locate

Peak fitting

Visual inspection of fitting

Bkg subtr. & Nuclide id.

Efficiency

Summing correction

Parent daughter correction

Reporting

Many steps => easy To get one wrong

Take care when using Automatic systems

Non-automatic => Problems with integrity of data

Uncertainty? Often only counting statistics

European Commission

$$A = \frac{C_{TOT} - C_{Peak}^{Bkg} - C_{Continuum}}{\varepsilon_{REF}^{Exp} \frac{\varepsilon_{Sample}^{MC}}{\varepsilon_{REF}^{MC}} P_\gamma} e^{\lambda t_d} \frac{\lambda}{(1 - e^{-\lambda t_m})} K_1 K_2 K_3$$

K_1 = summing correction

K_2 = Branching correction

K_3 = Equilibrium correction

Combine activity from several gamma-rays to activity for one radionuclide

Combine activity from several daughters to one parent

Ex.: ^{226}Ra from ^{214}Bi and ^{214}Pb